WiSe 2016/17 Semantics 1

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Atomic and Complex Formulae

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Homework

- Our literary scenario: see the links in the wiki: https://www.lexical-resource-semantics.de/wiki/index.php/Semantics_1,_WiSe_2016/17_(Sailer)
- ~~Read Levine et al., Chapter 2, Section 2~~.
- Atomic formulae: Using your model from this week's homework,
  - > Give 2 atomic formulae (one true, one false)
  - > ~~Give 2 statements with 1 connective each. (Use different connectives!)~~
  - > Provide the step-by-step computation of the truth of your 2 atomic formulae.
- For the computation, watch the videos on the wiki page.
- ~~Find 2 naturally occuring uses of "and" combining two sentences. Is there an extra meaning in addition to requiring both sentences to be true?~~

[[kicks_2(luck, est)]] = true
- iff < [[luck]], [[est]] > is in [[kicks_2]]
iff < I(luck), I(est) > is in I(kicks_2)
iff < Lucky, Estragon > is in {<x,y> | x kicks y}
since this is the case
[[kicks_2(luck, est)]] = true

- shoots$_2$(Vlad, Ez)

[[shoots$_2$(Vlad, Ez)]] = true
iff < [[Vlad]], [[Ez]] > is in [[shoots$_2$]]
iff < I(Vlad), I(Ez) > is in I(shoots$_2$)
iff < e, I > is in {<x,y> | x shoots y}

Truth conditions

since this is wrong,
[[shoots$_2$(Vlad, Ez)]] = false

Evaluation

$shoot_2$

$I(shoot_2) = \{\langle x, y\rangle \mid x \text{ shoots } y\}$
$= \{\ \} = \emptyset$

$female_1$

$I(female_1) = \{\langle x\rangle \mid x \text{ is female}\}$
$= \emptyset$

$female_1(vlad)$

$\cancel{female_1(vlad, esta)}$ not a formula!

waiting-for-Godot$_1$

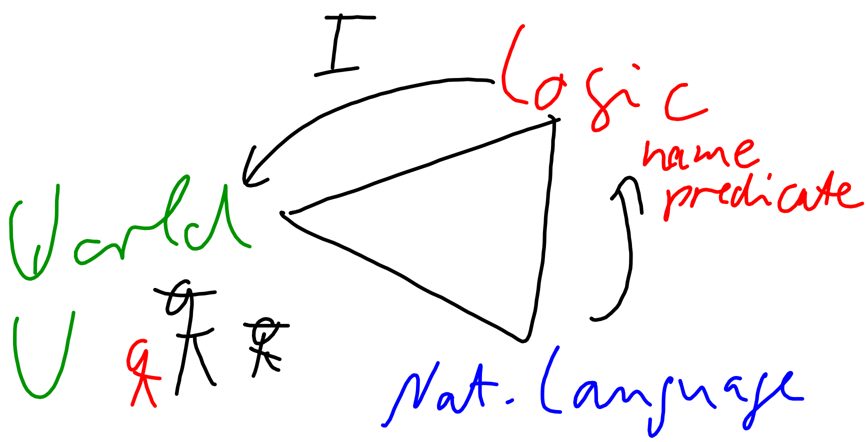
$I(\text{w-f-G}_1) = \{ \langle x \rangle \mid x \text{ is waiting for Godot} \}$

$= \{ \langle \ \rangle, \langle \ \rangle \}$

w-f-G$_1$(vlad)

eat$_1$

w-f-G$_1$(estra)



Waiting for Godot

Characters:

Vladimir

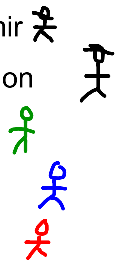
Estragon

Pozzo

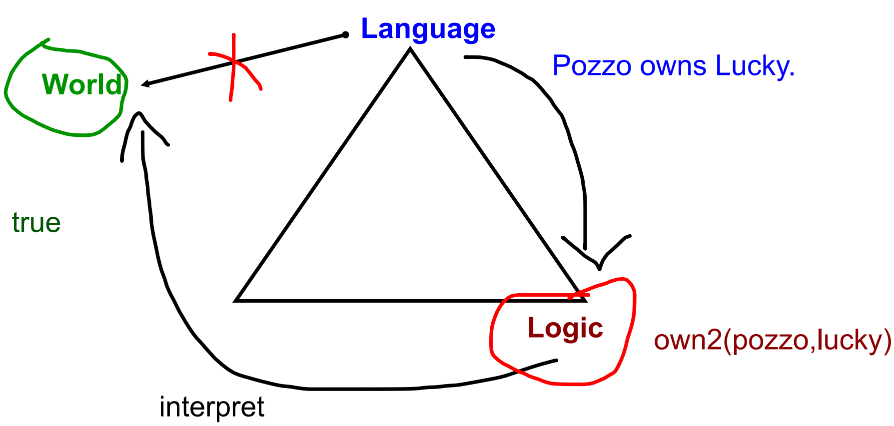
Lucky

A boy

(Godot)



Three levels



translation

I

Language | Logic | World

proper noun | name symbol | individual

Eshajm Vladimir | esha vlad

I

VP, common noun, adjective, ... | predicate symbol | relation/property

hug smaller than | hug_2 want-to-p.ty

$\{\langle x, y\rangle \mid \ldots\}$

denotation $\{\langle x\rangle \mid \ldots\}$

sentence | formula predicate symbol+right number of name symbos | truth values (proposition)

E. hugged V. | $hug_2(esha, vlad)$ | true false

Recipe for atomic formulae

1. Take a predicate symbol.
2. Look at its arity (i.e. the little number subscript)
3. Take the number of name symbols that correspond to the arity.
4. Write down: predicate-symbol_n(name1, ..., name_n)

Interpreting formulae: Denotation function

[[...]] depends on the model M = < U, I >

Denotation of a name:

Denotation of a predicate symbol:

Denotation of an atomic formula:

Computing the truth value of an atomic formula

1. Determine the truth conditions
2. Evaluate them with respect to our model.

Step 1: Determine the truth conditions

[[homeless1(estragon)]] = true

if and only if (iff) < [[estragon]] > is in [[homeless1]]

iff < I(estragon) > is in I(homeless1)

iff <  > is in { <x> | x is homeless } = { <  > , <  > }

Step 2: Evaluate them in our model

Since this is the case, [[homeless1(estragon)]] = true.

Complex formulae: Connectives

[$\overbrace{\text{Lucky is Pozzo's servant}}^{\varphi}$] and [$\overbrace{\text{Estragon is Vladimir's friend}}^{\psi}$]

Translate the components

φ : servant-of$_2$(luck, pozz) | ψ : frend-of$_2$(estr, vlad)

Truth value of the components

$[\![\varphi]\!]$ = true $[\![\psi]\!]$ = true

Truth value of the whole?

Logical connectives

and: ∧ &

or: ∨ or

not: ¬ ~

if... (then)...: ⊃ →

Conjunction and

Truth table:

φ	ψ	φ∧ψ	Example sentence
T	T	T	C is P's servant and V. is E's friend.
T	F	F	C is P's servant and P. is C's servant.
F	T	F	P is waiting for Godot and E is V's friend.
F	F	F	C is rich and P is poor.

Truth conditions:

[[φ∧ψ]] = true iff [[φ]] = true and [[ψ]] = true.

Everyday experience with and

Für Akademiker und Sies mit Nivea.

She's a doctor and he's a football player.

~~She~~ Knocked at the door and ~~she~~ He entered.

He entered and knocked at the door.

He didn't learn for the test temporal order

and he failed. causal relation

He knocked at the door. He entered.

He entered. He knocked at the door.

"Extra meaning component" is a discourse effect!

1) and can be used without extra meaning component.

2) we find the extra meaning component also without and.

Disjunction or ("inclusive or")

φ	ψ	$\varphi \vee \psi$	
T	T	T	E. is vain for G. or P. is rich.
T	F	T	E. is vain for G. or L. is rich.
F	T	T	L. is rich or E. is vain for G.
F	F	F	L. is rich or P. is poor

$[\![\varphi \vee \psi]\!] = true$ iff $[\![\varphi]\!] = true$ or $[\![\psi]\!] = true$ or both.

inclusive or — exclusive or

Do you take milk or sugar? — Do you take tea or coffee?

Negation

NOT ¬ "it is not the case that ..."

For each formula φ, [[¬φ]]= true iff

φ	¬φ	
T	F	It is not the case that E is vain for G.
F	T	It is not the case that V is not hungry.

$[\![\neg \varphi]\!] = true$ iff $[\![\varphi]\!] = false$

"The law of the excluded middle"

$[\![\neg\, v\text{-}f\text{-}G_1(lnd)]\!] = true$

iff $[\![v\text{-}f\text{-}G_1(lnd)]\!] = false$

iff $\langle [\![lnd]\!] \rangle$ is not in $[\![v\text{-}f\text{-}G_1]\!]$

iff $\langle I(lnd) \rangle$ is not in $I(v\text{-}f\text{-}G_1)$

iff $\langle\ \rangle$ is not in $\{\langle x \rangle \mid x$ is vain for G.$\}$

$= \{\langle\ \rangle, \langle\ \rangle\}$

Since this is the case, $[\![\neg\, v\text{-}f\text{-}G_1(lnd)]\!] = true$

$[\![\text{homeless}_1(\text{laura}) \vee \text{v-f-G}_1(\text{vlad})]\!] = \text{true}$

iff $[\![\text{homeless}_1(\text{laura})]\!] = \text{true}$ or $[\![\text{v-f-G}_1(\text{vlad})]\!] = \text{true}$ or both

iff $\langle [\![\text{laura}]\!] \rangle$ is in $[\![\text{homeless}_1]\!]$ or $\langle [\![\text{vlad}]\!] \rangle$ is in $[\![\text{w-f-G}_1]\!]$ or both

iff $\langle I(\text{laura}) \rangle$ is in $I(\text{homeless}_1)$ or $\langle I(\text{vlad}) \rangle$ is in $I(\text{w-f-G}_1)$ or both

iff $\langle \ \rangle$ is in $\{\langle x \rangle \mid x \text{ is homeless}\}$ or $\langle \ \rangle$ is in $\{\langle x \rangle \mid x \text{ is v.f.G.}\}$

Since the first disjunct is false but the second disjunct is true, the overall formula is true.

For next week

- Read Levine et al., Chapter 2, Section 2.
- Complex formulae: Using your model from this week's homework,
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- For the computation, watch the videos on the wiki page.
- Find 2 naturally occurring uses of "and" combining two sentences. Is there an extra-meaning in addition to requiring both sentences to be true?
- Find 2 naturally occurring uses of "or" combining two sentences. Is there an extra-meaning in addition to requiring that at least one sentence be true?